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MTH 530, Abstract Algebra I (graduate) Fall 2012 , Final Exam

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QUESTION 1. Let G be a group such that $a^2 = e$ for each $a \in G$. Prove that G is Abelian.

QUESTION 2. Let a be an element in a group G such that $a^n = e$ for some positive integer n. If m is a positive integer such that $gcd(n, m) = 1$, then prove that $a = b^m$ for some b in G.

QUESTION 3. Prove that $U(20)$ is of rank 2. Show the work.

QUESTION 4. Let G be an infinite group. Prove that G has infinitely many distinct proper subgroups.

QUESTION 5. Let $\alpha, \beta \in S_5$ such that α is a 5-cycle and β is a 2-cycle. Given $|\alpha \ o \ \beta|$ is strictly less than $|\alpha^2 \ o \ \beta|$. Find $|\alpha^3 \ o \ \beta \ o \ \alpha^4|$ and $|\beta \ o \ \alpha^4|$.

QUESTION 6. Let f be a group homomorphism from A_5 into A_4. Prove that f is the trivial group-homomorphism.

QUESTION 7. Let $n \geq 3$ and D be a subgroup of S_n such that $|D| = n!/2$. Prove that $D = A_n$.

QUESTION 8. Let F be an abelian group of order p^n for some $n \geq 2$ such that F has a unique subgroup of order p^k for some k, $1 \leq k < n$. Prove that F is cyclic.

QUESTION 9. Let F be a group of order 80. Prove that F is not simple.

QUESTION 10. Prove that S_7 does not have a subgroup of order 15

QUESTION 11. i) Prove that A_5 has a subgroup of order 12 but not a subgroup of order 20. ii) Let F be a simple group of order 60 and assume that F has a subgroup of order 12. Prove that F is isomorphic to A_5

QUESTION 12. i) Given F and G are abelian finitely generated groups such that $|G| = |F| < \infty$ and $Rank(F) = Rank(G)$. Can we conclude that F is isomorphic to G? Prove or disprove.

ii) Given D is an infinite finitely generated abelian group of rank 4. Let M be the torsion part of D. Given $|M|$ = 32 and M has exactly 2 subgroups of order 4.. find all non-isomorphic possibilities for the group D. [Hint: here is one place where two contradictions stated back to back!!!]

Faculty information

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